Transient Sub-Poissonian Distribution for Single-Mode Lasers

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Abstract In this paper, the transient photon statistics for single-mode lasers is investigated by making use of the theory of quantum electrodynamics. By taking into account of the transitive time τ, we obtain the master equation for Jaynes-Cummings model. The relation between the Mandel factor and the time is obtained by directly solving the master equation. The result shows that a transient phenomenon from the transient super-Poissonian distribution to the transient sub-Poissonian distribution occurs for single-mode lasers.

In addition, the influences of the thermal light field and the cavity loss on the transient sub-Poissonian distribution are also studied.

Key words: single-mode laser; Jaynes-Cummings model; Transient sub-Poissonian photon statistics.

1 Introduction

As is well known, sub-Poissonian light field is a typical nonclassical light field. And it has widely applications to the ultraweak signal detection and to the optical communication etc.[1] According to the usual theory, there is no sub-Poisonian distribution for single-mode lasers.

In this paper, the transient photon statistics for single-mode lasers is investigated by making use of the theory of quantum electrodynamics. by taking into account of the transitive time τ. the master equation for Jaynes-Cummings model and its solution are obtained.

2 Master equation

First of all, the interaction of one atom with the light field is taking into account. According to the theory of the quantum electrodynamics. for the Jaynes-Cummings model the Hamiltonion has the following form[2-4](with $\frac{h}{2\pi}=1$)

$$H=\omega a^{+}a+\frac{1}{2}\omega_0\sigma_z+g(a\sigma^{+}+a^{+}\sigma^{-}), \tag{1}$$

where a and a^{+} are annihilation and creation operators of photon; σ^{+} and σ^{-} are raising and lowering operators of the atom; ω and ω_0 are the mode frequency and the transition frequency, respectively; σ_z is the inversion papurition of the atom; g is the coupling constant between the atom and the field mode.

The eignequation of the expcession (1) is given by

$$H|\Phi\rangle=E|\Phi\rangle, \tag{2}$$

where

$$E_n^{\pm}=[\omega(n+\frac{1}{2})\pm\Omega_n] \tag{3}$$

$$Eg=-\frac{1}{2}\omega_0 \tag{4}$$

and

$$\Omega_n=[(\frac{\triangle}{2})^2+g^2(n+1)]^{\frac{1}{2}} \tag{5}$$

$$\triangle=\omega-\omega_0 \tag{6}$$

The eignstates corresponding to expressions (3) and (4) are given by

$$|\Phi_n^{\pm}\rangle=\begin{pmatrix}\sin\theta_n\\ \cos\theta_n\end{pmatrix}|n,a\rangle\pm\begin{pmatrix}\cos\theta_n\\ \sin\theta_n\end{pmatrix}|n+1,b\rangle \tag{7}$$

$$|\Phi g\rangle=|0,b\rangle \tag{8}$$

here

$$\theta_n = \tan^{-1}\left(\frac{g\sqrt{n+1}}{\frac{\Delta}{2}+\Omega_n}\right) \tag{9}$$

where n denoting the photon number; a and b denoting the upper and lower atomic levels.

All nonzero matrix elements of the evolving operator

$$U(\tau) = \exp(-iH\tau) \tag{10}$$

in the state $|n, \alpha\rangle = |n\rangle|\alpha\rangle$ ($\alpha = a, b$) are given by

$$a_n = \langle n+1, b|U(\tau)|n+1, b\rangle = \cos^2\theta_n e^{-iE_n^+\tau} + \sin^2\theta_n e^{-iE_n^-\tau} \tag{11}$$

$$b_n = \langle n+1, b|U(\tau)|n, a\rangle = \sin\theta_n\cos\theta_n(e^{-iE_n^+\tau} - e^{-iE_n^-\tau}) \tag{12}$$

$$C_n = \langle n, a|U(\tau)|n, a\rangle = \sin^2\theta_n e^{-iE_n^+\tau} + \cos^2\theta_n e^{-iE_n^-\tau} \tag{13}$$

and

$$B_n(\tau) = |b_n(\tau)|^2 = \frac{g^2(n+1)}{\frac{\Delta}{2}+g^2(n+1)}\sin^2\left(\sqrt{\left(\frac{\Delta}{2}\right)^2+g^2(n+1)}\,\tau\right), \tag{14}$$

Assuming at the initial time t there is no correlation between the atom and the fidd, thus we have

$$\rho_s(t) = \rho_a(t) \otimes \rho(t). \tag{15}$$

This means that the matrix elements of $\rho_s(t)$ is the combination state $|n, \alpha\rangle$ and can be written as

$$\langle n, \alpha|\rho_s(t)|n', \alpha'\rangle = \langle n|\rho(t)|n'\rangle\langle\alpha|\rho_a(t)|\alpha'\rangle. \tag{16}$$

After τ, the expression (15) becomes

$$\rho_S(t+\tau) = U(\tau)\rho_s(t)U^{-1}(\tau) \tag{17}$$

and

$$\rho(t+\tau) = \sum_\alpha \langle\alpha|\rho_s(t+\tau)|\alpha\rangle \tag{18}$$

In the photon number representation, the matrix elements of equation (18) may be given by

$$\rho_{n,m}(t+\tau) = \sum_k\sum_{k'} \langle n, m|G(\tau)|k, k'\rangle\langle k|\rho(t)|k'\rangle, \tag{19}$$

where

$$<n, m|G(\tau)|k, k'> = \sum_{\alpha}\sum_{\alpha'}\sum_{\alpha''} <n, \alpha''|U(\tau)|k, \alpha><k', \alpha'|U^{-1}(\tau)|m, \alpha''> P_{\alpha\alpha'}, \tag{20}$$

where $|m>$ and $|k>$ denotes the photon-number states, and

$$P_{\alpha\alpha'} = <\alpha|\rho_a(t)|\alpha'>. \tag{21}$$

For the arbitrary initial state of the atom and the light field, using expressions (11)—(13), (20) and (21), we obtain

$$\begin{aligned}\rho_{n,m}(t+\tau) = {} & P_{aa}[a_n a_m{}^* \rho_{n,m}(t) + b_{n-1} b^*_{m-1} \rho_{n-1,m-1}(t)] \\ & + P_{ab}[b_n a_m{}^* \rho_{n+1,m}(t) + C_{n-1} b^*_{m-1} \rho_{n,m-1}(t)] \\ & P_{ba}[a_n b_m{}^* \rho_{n,m+1}(t) + b_{n-1} C^*_{m-1} \rho_{n-1,m}(t)] \\ & P_{bb}[b_n b_m{}^* \rho_{n+1,m+1}(t) + C_{n-1} C^*_{m-1} \rho_{n,m}(t)]\end{aligned} \tag{22}$$

Expression (22) is a generol form. For the laser system under considern-tion, we have

$$P_{ab} = P_{ba} = 0 \tag{23}$$

By taking into account equation (14) and the following expression

$$|a_n|^2 + |b_n|^2 = |C_n|^2 + |b_n|^2 = 1 \tag{24}$$

then equation (22) can be deduced to the following form:

$$\begin{aligned}\rho_{n,m}(t+\tau) = {} & P_{aa}\{\sqrt{[1-B_n(\tau)][1-B_m(\tau)]}\rho_{n,m}(t) \\ & + \sqrt{B_{n-1}(\tau)B_{m-1}(\tau)}\rho_{n-1,m-1}(t)\} + P_{bb}\{\sqrt{B_n(t)B_m(t)} \\ & \cdot \rho_{n+1,m+1}(t) + \sqrt{[1-B_{n-1}(t)][1-B_{m-1}(t)]}\rho_{n,m}(t)\}.\end{aligned} \tag{25}$$

Under the coarse grain approximation, equation of motion for the density matrix elements are given by

$$\rho_{n,m}(t) = \nu\int_0^{\tau} d\tau' P(\tau')[\rho_{n,m}(t+\tau') - \rho_{n,m}(t)] + L\rho_n, m(t), \tag{26}$$

where

$$p(\tau') = Ne^{-\nu\tau'} \tag{27}$$

denotes the distribution function of the interaction duration τ between atom and field; N is a normalization constant; ν stands for the atomic decay rate.

Form the norwalization condition

$$\int_0^\tau P(\tau')d\tau' = 1 \tag{28}$$

we get

$$N = \frac{\nu}{1-e^{-T}} \tag{29}$$

where

$$T = \nu\tau \tag{30}$$

By substituting expression (25) into (26) and making use of Ref. [5], we finally obtain

$$\begin{aligned}
\dot{\rho}_{n,m} = &- \nu_a \rho_{n,m}(t)\{1 - \int_0^\tau d\tau' P(\tau') \sqrt{[1 - B_n(\tau')][1 - B_m(\tau')]}\} \\
&+ \nu_a \rho_{n-1,m-1}(t) \int_0^\tau d\tau' P(\tau') \sqrt{B_n(\tau') B_{m-1}(\tau')} \\
&- \nu_b \rho_{n,m}(t)\{1 - \int_0^\tau d\tau' P(\tau') \sqrt{[1 - B_{n-1}(\tau')][1 - B_{m-1}(\tau')]}\} \\
&+ \nu_b \rho_{n+1,m+1}(t) \int_0^\tau d\tau' P(\tau') \sqrt{B_n(\tau') B_m(\tau')} \\
&- \frac{c}{2} n_b [(n + 1 + m + 1)\rho_{n,m}(t) - 2\sqrt{nm}\,\rho_{n-1,m-1}(t)] \\
&+ \frac{c}{2} n_b [2\sqrt{(n + 1)(m + 1)}\rho_{n+1,m+1}(t) - (n + m)\rho_{n,m}(t)]
\end{aligned} \tag{31}$$

where n_b is the average photon number of the thermal light field; C is the cavity loss.

Expression (31) is the master equation for the single-mode lasers.

3 Numerical calculation

In the case of resonance, master equation (31) can be reduced to the following form:

$$\dot{\rho}_{n,m}(t) = - \nu_a [1 - (A^-_{n,m} + A^+_{n,m})]\rho_{n,m}(t)$$

$$+\nu_a[(A^-_{n-1,m-1}-A^+_{n-1,m-1}]\rho_{n-1,m-1}(t)$$
$$-\nu_b[1-(A^-_{n-1,m-1}+A^+_{n-1,m-1})]\rho_{n,m}(t)$$
$$+\nu_b[A^-_{n,m}-A^+_{n,m}]\rho_{n+1,m+1}(t)$$
$$-\frac{c}{2}n_b[(n+1+m+1)\rho_{n,m}(t)-2\sqrt{nm}\,\rho_{n-1,m-1}(t)]$$
$$+\frac{c}{2}n_b[2\sqrt{(n+1)(m+1)}\,\rho_{n+1,m+1}(t)-(n+m)\rho_{n,m}(t) \quad (32)$$

where

$$A^{\pm}_{n,m}=\frac{1}{2}\int_0^{\tau}d\tau' P(\tau')\{\sqrt{[1-B_n(\tau')][1-B_m(\tau')]}\mp\sqrt{B_n(\tau')B_m(\tau')}\}=\frac{1}{2(1-e^{-T})}\cdot$$
$$\frac{1+e^{-T}\{A(\sqrt{n+1}\pm\sqrt{m+1})\sin[A(\sqrt{n+1}\pm\sqrt{m+1})T]-\cos[A(\sqrt{n+1}\pm\sqrt{m+1})T]\}}{1+[A(\sqrt{n+1}\pm\sqrt{m+1})]^2} \quad (33)$$

In particulnr, for the diagonal matrix elements[5] expression (32) may be further reduced to the following form:

$$\dot{P}_n(\sigma)=\dot{\rho}_{n,n}(\sigma)=-(\frac{1}{2}+\frac{\mu}{2}+\frac{(2n+1)n_b+n}{R}-A^+_n-\mu A^+_{n-1})\rho_{n,n}(\sigma)$$
$$+(\frac{1}{2}-A^+_{n-1}+\frac{n-n_b}{R}\rho_{n-1,n-1}(\sigma)+(\frac{\mu}{2}-\mu A^+_n+\frac{(n+1)(n_b+1)}{R})\rho_{n+1,n+1}(\sigma) \quad (34)$$

where

$$A^+_n=\frac{1}{2(1-e^{-T})}\frac{1+e^{-T}\{\frac{2\chi}{\sqrt{2R}}\sqrt{n+1}\sin[\frac{2\chi t}{\sqrt{2R}}\sqrt{n+1}]-\cos[\frac{2\chi t}{\sqrt{2R}}\sqrt{n+1}]\}}{1+[\frac{2\chi}{\sqrt{2R}}\sqrt{n+1}]^2} \quad (35)$$

here $R=\dfrac{\nu_a}{C}$,

$\mu=\dfrac{A_b}{A}$,

$\chi=\sqrt{\dfrac{A}{C}}$,

$\sigma=\nu_a t$;

and

$$A = 2\nu_a\left(\frac{g}{\nu}\right)^2, \tag{37}$$

$$A_b = 2\nu_b\left(\frac{g}{\nu}\right)^2.$$

The photon statistical properties of the lingt field can be expressed by Mandel factor Q:

$$Q = \frac{<n^2>-<n>^2-<n>}{<n>}, \tag{38}$$

where

$$<n>= \sum_{n=0}^{\infty} nP_n,$$

$$<n^2>= \sum_{n=0}^{\infty} n^2P_n, \tag{39}$$

During the trnnsient processes, Mandel factor $Q>0$, $Q=0$ or $Q<0$ correspond to transient super-Poissonian distribution, Poissonian distribultion or sub-Poissonian distribution, respectively.

Time evolution of the Mandel factor may be obtained by making use of the expressions (34), (35) ,(38) and (39). The numerical results are shown in Figures 1-5.

Figure 1 shows that the transient photon statistical property passes from super-Poissonian distribution through Poissonian distribution into sub-Poissoninn distribution with the increase of σ.

Figure 2 shows that the maximum value of the Q drift apart from the right and decrease. At the same time, the velocity toward the trnnsient sub-Poissonian distribution is also quickened.

Figure 3 indicates that the influence of the loss μ on the Mandel factor is marked and the transient sub-Poissonian distribution will disappear when the χ increase to some certain value.

Figure 4 indicates that the thermal light photon number not only deerease sub-Poissonian distribution but also diminish the velocity for toward sub-Poissonian distribution.

4 Brief discussion

In the present paper, we have studied the transient sub-Poissonian distribution for single-mode lasers. The result shows that for single-mode lasers the sub-Poissonian distribution may occur not only in the case of stotionary state[5] but also in the case of transient state.

As is well known, transient sub-Poissonian photon statistics is a character for the quantum light field. And its appearance would deepen our knowledge of the light field essence.

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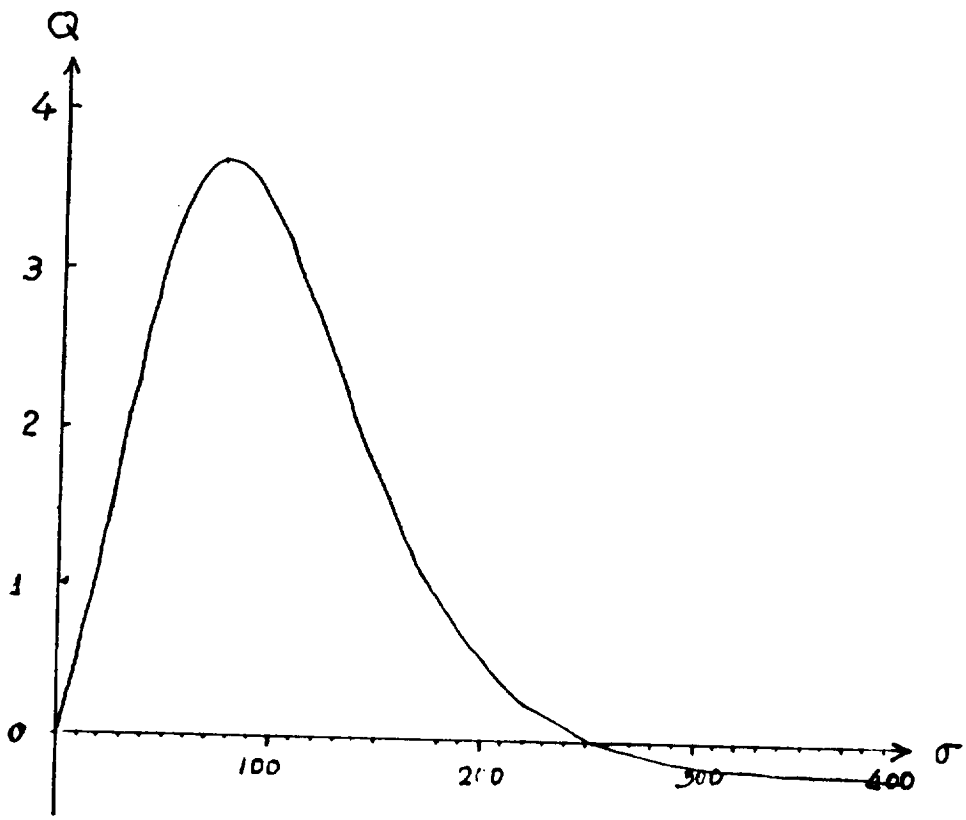

Fig. 1.

Caption of Fig. 1. Time evolution of the mandel factor for $T=1.1$; $R=100$; $\mu=n_b=0$, $\chi=6$

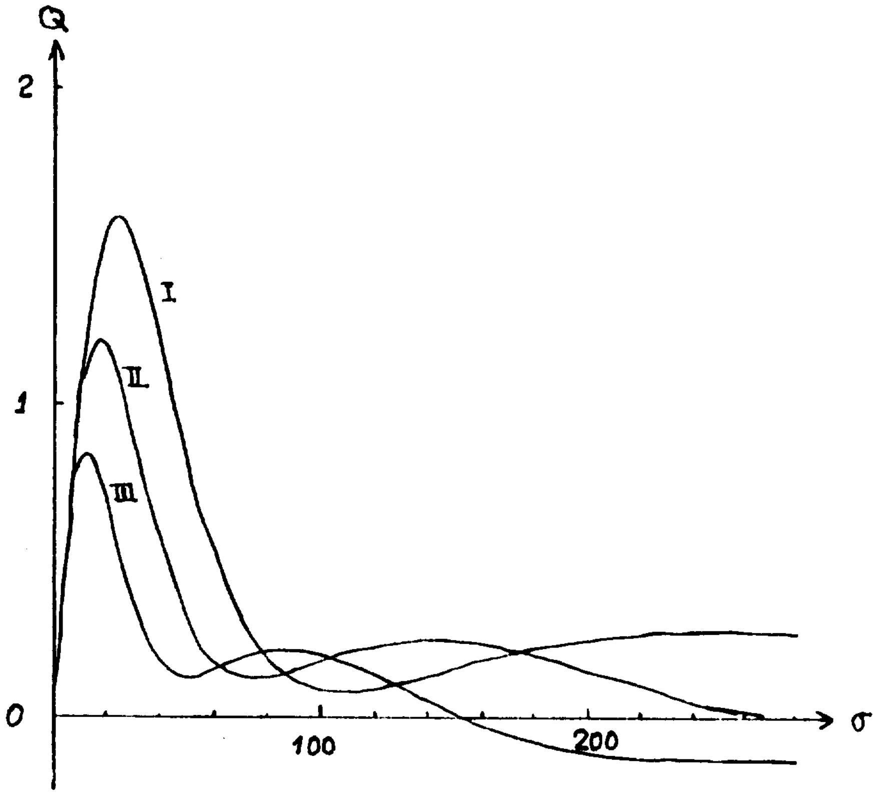

Fig. 2.

Caption of Fig. 2. Time evolution of the mandel factor for $T=1.1$; $R=100$; $\mu=n_b=0$, (I) $\chi=9.5$; (II) $\chi=10.8$; (III) $\chi=12.5$

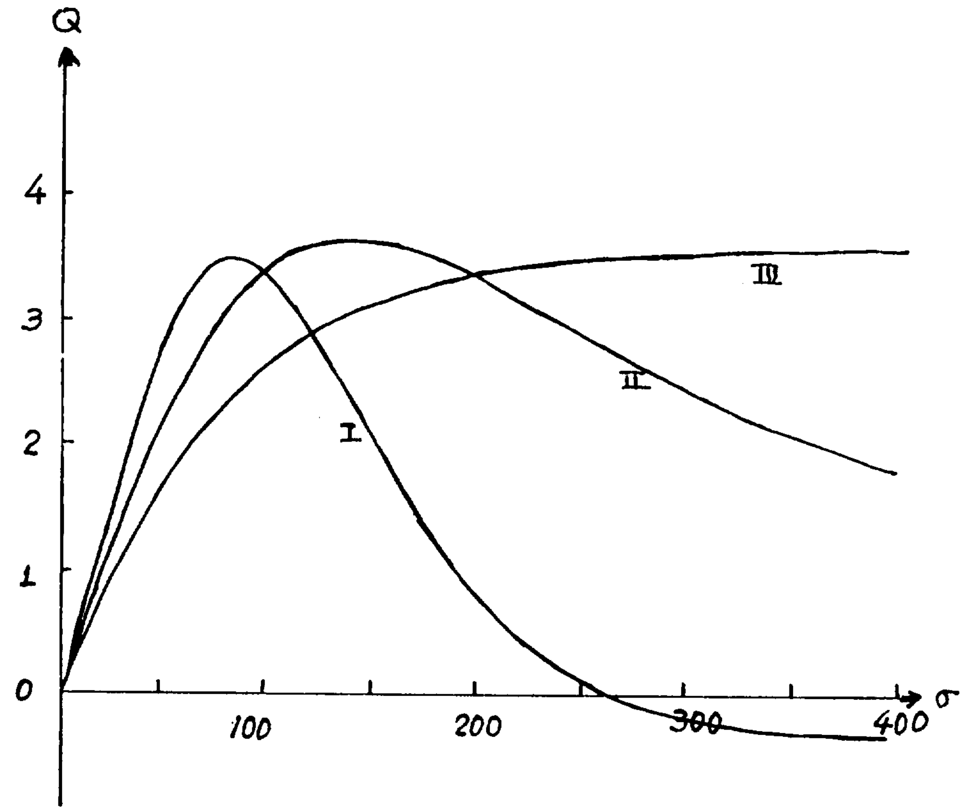

Fig. 3.

Caption of Fig. 3. Time evolution of the mandel factor for $T=1.1$; $R=100$; $n_b=0$; $\chi=6$; (I) $\mu=0.1$; (II) $\mu=0.5$; (III) $\mu=0.8$

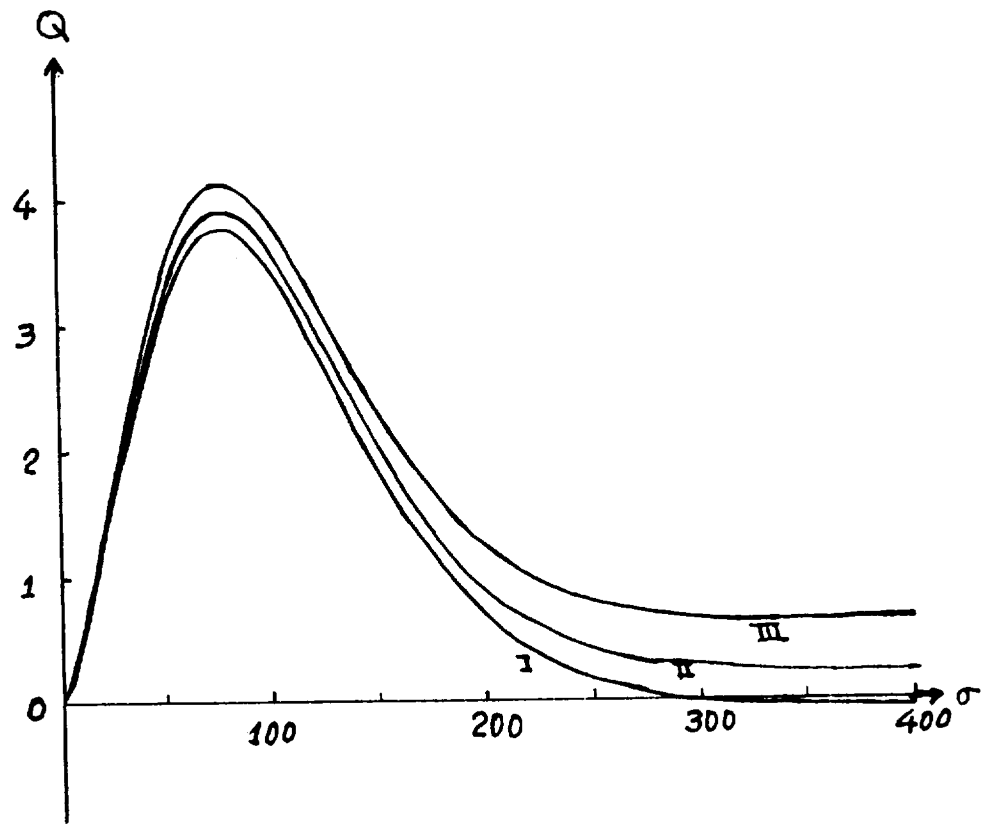

Fig. 4.

Caption of Fig. 4. Time evolution of the Mandel factor for $T=1.1$; $\chi=6$; $R=100$; $\mu=0$; (I) $n_b=0.2$; (II) $n_b=0.5$ (III) $n_b=1$